328. **Proposed by W. J. GREENSTREET, M. A., Marling School, Stroud, England.**

If $x^2+xy+y^2=3a^2$, find the maximum value of $bx+cy$.

Solution by G. B. M. ZERR, A. M., Ph. D., Philadelphia, Pa., and J. SCHEFFER, A. M., Hagerstown, Md.

$x^2+xy+y^2=3a^2$, $bx+cy$=maximum.

$$\therefore \frac{dx}{dy}=-\frac{x+2y}{2x+y}=-\frac{c}{b}, \text{ or } y=\frac{(2c-b)x}{2b-c}.$$

$$\therefore x^2=\frac{a^2(2b-c)^2}{b^2-bc+c^2}; \text{ and } x=\pm\frac{a(2b-c)}{\sqrt{(b^2-bc+c^2)}},\ y=\pm\frac{a(2c-b)}{\sqrt{(b^2-bc+c^2)}}.$$

$$\therefore bx+cy=2a\sqrt{(b^2-bc+c^2)}=2a\sqrt{\frac{b^3+c^3}{b+c}} \text{ is a maximum.}$$

Also solved by F. L. Griffin and S. G. Barton.

329. **Proposed by C. N. SCHMALL, 604 East 5th Street, New York City.**

Between the quantities a and b there are inserted n arithmetical and n harmonical means, and a series of n terms is formed by dividing each arithmetical by the corresponding harmonical mean. Show that the sum of the series is, $n\left[1+\frac{n+2}{n+1}\cdot\frac{(a-b)^2}{6ab}\right]$.

Solution by HOWARD C. FEEMSTER, A. B., Professor of Mathematics, York College, York, Neb., and S. G. BARTON, Ph. D., Clarkson School of Technology, Potsdam, N. Y.

The n arithmetic means between a and b are:

$$(1)\ \frac{b+na}{n+1},\ \frac{2b+(n-1)a}{n+1},\ \ldots,\ \frac{rb+(n-r+1)a}{n+1},\ \ldots$$

The n harmonic means between a and b are:

$$(2)\ \frac{ab(n+1)}{nb+a},\ \frac{ab(n+1)}{(n-1)b+2a},\ \ldots,\ \frac{ab(n+1)}{(n-r+1)b+(r-1)a},\ \ldots$$

Dividing the terms of (1) by the corresponding terms of (2),

$$\frac{(b+na)(a+nb)}{ab(n+1)^2},\ \frac{[2b+(n-1)a][(n-1)b+2a]}{ab(n+1)^2},\ \ldots,$$

$$\frac{[rb+(n-r+1)a][(n-r+1)b+2a]}{ab(n+1)^2}, \ldots, \text{ the series to be summed.}$$

$$\text{Hence } S=\frac{ab(n^2+1)+n(a^2+b^2)}{ab(n+1)^2}+\frac{ab[(n-1)^2+r^2]+2(n-1)(a^2+b^2)}{ab(n+1)^2}+\ldots$$

$$+\frac{ab[(n-r+1)^2+r^2]+r(n-r+1)(a^2+b^2)}{ab(n+1)^2}+\ldots$$

$$=\frac{1}{ab\,(n+1)^2}\{ab[n^2+(n-1)^2+\ldots+1^2]+ab[1^2+2^2+\ldots+n^2]$$

$$(a^2+b^2)[n+2(n-1)+3(n-2)+\ldots+r(n-r+1)+\ldots+n]\}=$$

$$=\frac{1}{ab\,(n+1)^2}\left[2ab\,\frac{n(n+1)(2n+1)}{6}+(a^2+b^2)\left(\frac{n(n^2+1)}{2}-\frac{n(n^2-1)}{3}\right)\right]$$

$$=\frac{1}{ab\,(n+1)^2}\left[2ab\,\frac{n(n+1)(2n+1)}{6}+(a^2+b^2)\left(\frac{n(n+1)\ (n+2)}{6}\right)\right]$$

$$=\frac{n}{ab\,(n+1)^2}\left[\frac{6ab\ (n+1)^2}{6}-\frac{n^2+3n+2}{6}\ .\ 2ab+\frac{(n+1)(n+2)}{6}(a^2+b^2)\right]$$

$$=n\left[1+\frac{a^2-2ab+b^2}{6ab\,(n+1)^2}.(n+1)\ (n+2)\right]=n\left[1+\frac{n+2}{n+1}.\frac{(a-b)^2}{6ab}\right].$$

Solved similarly by G. B. M. Zerr, and J. Scheffer.

GEOMETRY.

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352. Proposed by G. I. HOPKINS, Professor of Astronomy, High School, Manchester, N. H.

Required, to construct the triangle, having given the base, vertical angle and sum of the altitude and the two remaining sides.

Solution by G. B. M. ZERR, A. M., Ph. D., Philadelphia, Pa.

Let $AB=a$ be the given base; ACB the given vertical angle; p=to the sum of the altitude and the remaining sides. On AB describe the seg-